Electron beam excitation of left-handed surface electromagnetic waves at artificial interfaces

Yu. O. Averkov,* A. V. Kats, and V. M. Yakovenko
A. Ya. Usikov Institute for Radiophysics and Electronics, Ukrainian National Academy of Sciences, 12 Acad. Proskura Str., 61085 Kharkov, Ukraine

(Received 8 October 2008; revised manuscript received 14 January 2009; published 6 May 2009)

In this Brief Report we present the theoretical analysis of excitation of the surface plasmon polaritons by a thin electron beam propagating in the vacuum gap separating a plasmalike medium (metal) from an artificial dielectric with negative magnetic permeability. We have obtained and discussed the dispersion relation for the vacuum-gap-localized waves for an arbitrary vacuum-gap width. We have shown that the interface-localized waves with the negative total energy flux can be excited.

DOI: 10.1103/PhysRevB.79.193402 PACS number(s): 41.60.Bq, 52.35.Fp

Nowadays a good deal of attention is focused on studies of the electromagnetic properties of artificial media, especially left-handed media (LHM), and combined structures made of the juxtaposed planar slabs of epsilon-negative (ENG) and mu-negative (MNG) media. These media sustain the propagation of electromagnetic waves (EMWs) possessing negative group velocity (i.e., their group velocity $\mathbf{v}_{\mathrm{gr}}$ is antiparallel to the phase velocity $\mathbf{v}_{\mathrm{ph}}$, $\mathbf{v}_{\mathrm{gr}} \uparrow\downarrow \mathbf{v}_{\mathrm{ph}}$).[1,2] Such waves are also referred to as *backward* or *left-handed waves* (LHWs).[3] The bulk LHWs demonstrate a number of unusual properties which are of great interest for different applications. As a matter of fact, there can possibly exist left-handed surface electromagnetic waves (LHSWs) that also possess exotic properties as their volume counterparts. In close analogy to the media supporting bulk LHWs and thus often called LHM, such interfaces could be called left-handed surface. The LHSWs are existent at different interfaces starting with a thin metallic layer sandwiched between two dielectric half spaces (see Ref. 4), supporting well-known surface plasmon polaritons (SPPs), and the most advanced interfaces including artificial media such as double negative and double positive ones (see Ref. 5), and ENG/MNG interfaces (see, e.g., Ref. 6 and references therein). The electrodynamic properties of ENG/MNG interfaces have been theoretically investigated in Ref. 7. Examination of surface EMWs properties and excitation of these waves are of considerable interest both from the scientific point of view and due to their possible role in subsequent miniaturization of the devices for light manipulation, which is currently a top-priority issue.

Surface wave excitation can be achieved by different methods, but if one of the neighboring media forming an interface that supports the surface EMWs is transparent, then excitation can be realized using optical methods (the total internal reflection in Otto or Kretschmann geometry and diffraction by appropriate periodic structures; see Ref. 8 and references therein). However, if both the media are opaque, say, ENG/MNG interface, then surface waves cannot be excited by the above methods.

In the present Brief Report we deal with this particular case and analyze the excitation of left-handed quasisurface waves by means of an electron beam instability. The electron beam is supposed to propagate within a very narrow vacuum gap separating the ENG and MNG media. This method is shown to be an effective one for surface waves excitation over the GHz frequency range and it is really difficult to indicate other appropriate methods for the case under study. Existence of a narrow gap between the media results in the eigenmodes of the system differing from the media interface surface EMWs. However, for rather thin vacuum gap these hybridized eigenmodes possess the negative energy flux and thus they are left handed (backward). It should be emphasized that as the gap vanishes, these modes strictly revert to the surface waves of the ENG/MNG media interface.

We consider the simplest case where the wave vector $\vec{q}$ of the EMWs excited is parallel to the beam velocity $\vec{v}_0$, $\vec{q} \parallel \vec{v}_0$. Below it will be shown that in this instance the TM-polarized waves can only be excited. Note that the excitation of negative group-velocity TM- and TE-polarized surface waves at a vacuum/LHM interface by an electron bunch over the GHz frequency range has been theoretically examined in Ref. 9.

The system under consideration is shown in Fig. 1. This three-layer system consists of half space 1 with negative dielectric permittivity $\varepsilon_1 < 0$ and positive magnetic permeability $\mu_1 > 0$ (say, plasmalike or metal-like medium), and artificial dielectric half space 3 with $\varepsilon_3 > 0$, $\mu_3 < 0$ separated by vacuum gap 2 of thickness h. The coordinate system is chosen so that the z axis is directed normal to the interfaces and media 1 and 3 correspond to $z \leq -h/2$ and $z \geq h/2$, respectively. Let an electron beam moves with velocity v_0 in the positive Ox direction in $z=0$ plane. We suppose that the beam is infinitely thin in Oz direction and infinitely wide in Oy direction. Such an approximation implies that we suppose the radiation wavelength, λ, to be much greater than the beam thickness. Examine the excitation of TM-polarized EMWs propagating along Ox axis with electric and magnetic field components $\vec{E}_\ell = (E_{x\ell}, 0, E_{z\ell})$ and $\vec{H}_\ell = (0, H_\ell, 0)$, where

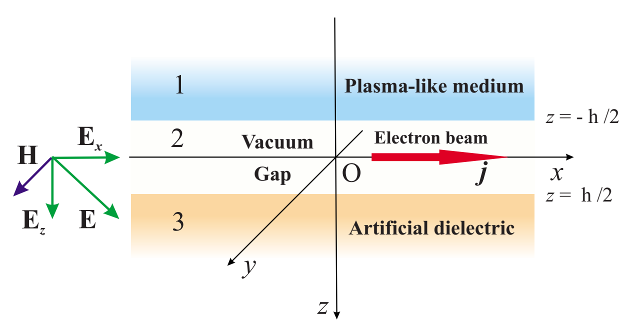

FIG. 1. (Color online) Geometry of the problem.

subindex ℓ, $\ell=1$, 2, and 3 denote the media. In half spaces 1 and 3 the EMW fields obey homogeneous Maxwell equations with regard to the time dispersion of the electromagnetic properties. We describe the fields in the electron beam using the Maxwell equations concurrently with the linearized continuity and motion equations in small-velocity and electron-density perturbations $v(x,t)$ and $n(x,t)$. Then the x component of the electron current-density perturbation is

$$j(x,z,t) = ev_0 n(x,t)\delta(z) + en_0 v(x,t)\delta(z), \tag{1}$$

where $\delta(z)$ is the Dirac delta function, e is the electron charge, and n_0 is the two-dimensional mean electron density. The latter can be expressed as $n_0=N_0 d$, where d is the beam thickness and N_0 denotes three-dimensional mean electron density. Let us seek for the solution of the problem in the exponential forms $n(x,t)$, $v(x,t)$, $\vec{E}_\ell$, and $H_\ell \propto \exp[i(qx-\omega t)]$. Then the electric displacement $\vec{D}_\ell$ and the magnetic induction $\vec{B}_\ell$ are $\vec{D}_\ell=\varepsilon_\ell(\omega)\vec{E}_\ell$ and $\vec{B}_\ell=\mu_\ell(\omega)\vec{H}_\ell$. The dispersion relation for the waves in the above system results from the system of Maxwell and linearized magnetohydrodynamic equations and continuity conditions for the tangential components of the electric and magnetic fields crossing the gap interfaces $z=\pm h/2$ along with the continuity conditions crossing the beam plane $z=0$. These are the continuity of the tangential component of the electric field and the jump of the tangential component of the magnetic field proportional to the current j.

The dispersion relation for the coupled TM waves is

$$\left[\frac{\Delta_{32}^{(+)}}{\Delta_{32}^{(-)}}\exp(-i\varphi) - \frac{\Delta_{12}^{(+)}}{\Delta_{12}^{(-)}}\exp(i\varphi)\right]\Omega^2 = \Lambda\left[1+\frac{\Delta_{12}^{(+)}}{\Delta_{12}^{(-)}}\exp(i\varphi)\right]\left[1+\frac{\Delta_{32}^{(+)}}{\Delta_{32}^{(-)}}\exp(-i\varphi)\right]. \tag{2}$$

Here $\Delta_{12}^{(\pm)}=Q_1/\varepsilon_1(\omega)\pm Q_2$, $\Delta_{32}^{(\pm)}=Q_3/\varepsilon_3(\omega)\pm Q_2$, $\Lambda=-i\omega_B^2 Q_2 d/2$, $\varphi=Q_2 h$, $\omega_B=\sqrt{4\pi e^2 N_0/m_0}$ is the beam electron plasma frequency,

$$\Omega = \omega - qv_0 \tag{3}$$

denotes the Doppler-shifted frequency, m_0 is the electron mass, and Q_l are the normal components of the wave vectors,

$$Q_\ell = \sqrt{\varepsilon_\ell(\omega)\mu_\ell(\omega)\omega^2/c^2 - q^2},$$

$$\mathrm{Re},\mathrm{Im}(Q_1) \le 0, \quad \mathrm{Re},\mathrm{Im}(Q_2) \ge 0, \quad \mathrm{Re},\mathrm{Im}(Q_3) \ge 0, \tag{4}$$

where c is the light velocity. The right-hand side in Eq. (2) is responsible for the interaction between the EMW modes of the empty vacuum-gap and the beam modes, $\Omega=0$. For $N_0 \to 0$ the interaction vanishes. Then vanishing of the first (second) multiplier on the left of Eq. (2) gives us the dispersion relation for the waveguide EMW eigenmodes (the electron-beam waves).

If the vacuum-gap thickness is far less than the wavelength, then $|\varphi| \ll 1$ and Eq. (2) can be simplified as

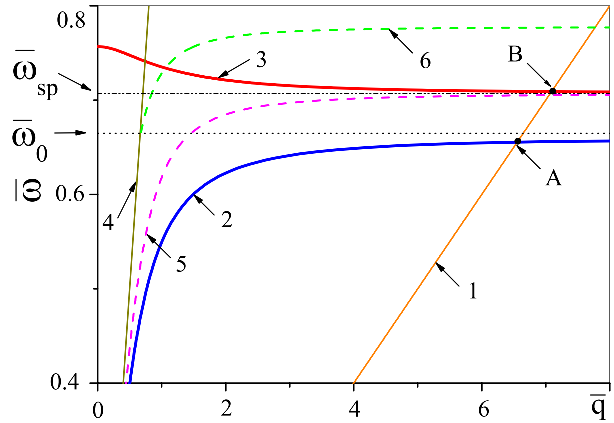

FIG. 2. (Color online) Dispersion curves of the surface and beam modes for vanishing interaction. Curves 5 and 6 correspond to the surface TM/TE waves existing at the metal-vacuum (vacuum-artificial dielectric) interfaces.

$$(\omega - qv_0)^2 \Delta_{TM}(\omega,q) = -i\omega_B^2 Q_1 Q_3 d, \tag{5}$$

where $\Delta_{TM}(\omega,q)=\varepsilon_3(\omega)Q_1-\varepsilon_1(\omega)Q_3$.

It should be noted that the surface TE modes are not excited by the beam for $\vec{q}\parallel\vec{v}_0$. The thing is that the electric field of the TE mode is perpendicular to the beam velocity $\vec{v}_0$ and, therefore, the velocity disturbance is perpendicular to $\vec{v}_0$ and thus the TE mode does not affect (in the linear approximation) the energy of the beam. Consequently, the energy exchange between the TE-polarized modes and the beam vanishes and the instability is evidently absent. The TE modes excitation is possible when $\vec{q}\nparallel\vec{v}_0$ or in high-order approximations for $\vec{q}\parallel\vec{v}_0$, but in these cases analytical calculations are more cumbersome. We will consider these cases in a subsequent paper.

For the numerical calculations we take $\varepsilon_1(\omega)$ and $\mu_3(\omega)$ in the form[10–12]

$$\varepsilon_1(\omega) = 1 - \frac{\omega_P^2}{\omega^2}, \quad \mu_3(\omega) = 1 - \frac{F\omega^2}{\omega^2-\omega_0^2}, \tag{6}$$

where ω_P is the metal plasma frequency, ω_0 is the resonance frequency of the artificial magnetic medium, and $F<1$ is the geometric form factor of this medium. Here we ignore the dissipation.

In Fig. 2 we depict the dispersion curves of the beam wave (curve 1) and the surface EMWs of the 1/3 interface (curves 2 and 3) with no interaction between EMWs and the beam wave. Curve 4 is the light line. Formally, the interaction vanishing corresponds to $\omega_B=0$. The specific parameters were chosen so that $\mu_1=\varepsilon_3=1$, $\bar{\omega}_0=\omega_0/\omega_P=0.66$, $F=0.56$, $\beta=v_0/c=0.1$, $\bar{\omega}=\omega/\omega_P$, and $\bar{q}=cq/\omega_P$ are the normalized frequency and the wave number. Points A and B denote the intersection of the straight line of the beam dispersion relation, $\omega=qv_0$, with dispersion curves 2 and 3 corresponding to the surface modes of the 1/3 interface. Here TM branch 2 exists in the frequency region where $\varepsilon_1(\omega)<-1$, $\mu_3(\omega)>0$. For $\bar{q}\to\infty$ its asymptote is the resonance frequency of magnetic permeability $\bar{\omega}\to\bar{\omega}_0$. TM branch 3 exists in the frequency region where inequalities $\varepsilon_1(\omega)<0$, $\mu_3(\omega)<0$, i.e.,

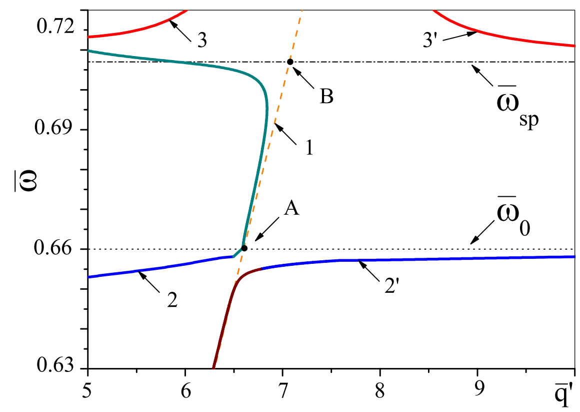

FIG. 3. (Color online) Detailed picture of the dispersion curves in close vicinities of the resonance points A and B.

$\bar{\omega}_0 < \bar{\omega} < \bar{\omega}_0/\sqrt{1-F}$, simultaneously hold true, and for $\bar{q}\to\infty$ it asymptotically approaches the surface plasmon frequency, $\bar{\omega}\to\bar{\omega}_{sp}=1/\sqrt{2}$. It is easy to show that the highest frequency relating to curve 3 (i.e., at $\bar{q}=0$) is $\bar{\omega}_{(-)}=(\sqrt{f^2+4\bar{\omega}_0^2F}-f)^{1/2}/(2F)^{1/2}\approx 0.756$, where $f=1-F$. The negative slope of dispersion curve 3 indicates that the absolute instability is possible in the interaction between the corresponding wave and the electron beam.[13] Corresponding primordial dispersion curves of TM modes (curve 5) and TE modes (curve 6) existing at the interfaces of vacuum/first medium and vacuum/third medium are shown, respectively. Dispersion curve 5 describes surface TM waves (the surface plasmon polaritons or SPP). It exists in the frequency region where the inequality $\varepsilon_1(\omega)<-1$ holds true. For $\bar{q}\to\infty$ curve 5 approaches the surface plasmon frequency from below, $\bar{\omega}\to\bar{\omega}_{sp}$. These waves are present in the frequency region defined by $\mu_3(\omega)<-1$, i.e., for $\bar{\omega}_0<\bar{\omega}<\bar{\omega}_0\sqrt{2/(2-F)}$. For $\bar{\omega}\to\bar{\omega}_0$ curve 6 approaches the light line at $\bar{q}=\bar{\omega}_0$. At $\bar{q}\to\infty$ it approaches the frequency $\bar{\omega}=\bar{\omega}_0/\sqrt{1-F/2}$ for which $\mu_3=-1$. It should be stressed that the artificial dielectric/metal interface supports two TM branches in contrast to the ordinary dielectric/metal interface, where the single low-frequency branch exists, $\omega<\omega_{sp}$ (see curve 5). Figure 3 presents the detailed structure of the splitting of initial dispersion curves in the close vicinities of the resonance points (i.e., close to the points A and B in Fig. 2).

Here $\bar{q}'=\mathrm{Re}(\bar{q})$. Curve 1 is for the electron-beam wave without interaction; curves 2 and 2′ are for the modes that result from the coupling of the low-frequency SPP mode and the beam mode (splitting near the point A in Fig. 2). In their turn curves 3 and 3′ correspond to the coupling of the high-frequency SPP mode (splitting near the point B in Fig. 2).

Let us concentrate on the vicinity of point B, where the SPP mode with negative energy flux could be generated by the electron beam. For $\omega=\omega_B+\delta\omega$, $|\delta\omega|\ll\omega$, $q=q_B+\delta q$, and $|\delta q|\ll q_B$, where (q_B,ω_B) are coordinates of point B,

$$\omega_B = q_B v_0, \quad \Delta_{TM}(\omega_B,q_B)=0, \tag{7}$$

the value $\delta\omega$ obeys the cubic equation [expansion of the dispersion relation in Eq. (5)] with real coefficients if the absorption is neglected. That is, we obtain one real and two complex conjugate roots, and one of the complex roots corresponds to the beam-induced instability with the increment

$$\omega_B'' \simeq \frac{\sqrt{3}}{2}\left[\frac{\omega_B^2\omega_P q d}{2(1+\varepsilon_3)^{3/2}}\right]^{1/3} \ll \omega_P. \tag{8}$$

As can be seen from Fig. 3, in the vicinity of point B, the dispersion curve with negative group velocity vanishes. This means that the corresponding root of the dispersion relation becomes complex and thus the beam instability occurs for the wave with the negative group velocity. The dispersion curves shown in the Fig. 3 in this region correspond to the stable eigenmodes with real values of $\delta\omega$.

Let us show that the total energy flux relating to the high-frequency modified SPP mode can remain negative even if it becomes unstable. This holds in the vicinity of point B, where the magnetic permeability of the artificial dielectric is negative, $\mu_3(\omega)<0$, and the dielectric permittivity of the metal $-\varepsilon_3<\varepsilon_1(\omega)<0$. The sum of the partial energy fluxes in 1 ($\Pi_1<0$) and 3 ($\Pi_3>0$) media for a thin vacuum gap ($|\varphi|\ll 1$) is as follows:

$$\Pi_1+\Pi_3 = \frac{c^2}{16\pi}|H_1^{(0)}(\omega,q)|^2 \times \mathrm{Re}\left\{-\frac{q}{\omega\varepsilon_1(\omega)Q_1''}+\frac{q}{\omega\varepsilon_3(\omega)Q_3''}|1+\eta_0|^2\right\} \times \exp(2\omega'' t)+O(hQ_2''), \tag{9}$$

where $Q_\ell'=\mathrm{Re}(Q_\ell)$, $Q_\ell''=\mathrm{Im}(Q_\ell)$, $\ell=1,2,3$, $\mathrm{Im}(\omega)=\omega''$, $H_1^{(0)}(\omega,q)$ denotes an initial amplitude of the magnetic field in medium 1, $\eta_0=-i\omega_B^2Q_1d/[\varepsilon_1(\omega)\Omega^2]$, and we have neglected a small absorption. The dimensionless parameter η_0 describes the electron-beam input into the relation between the magnetic-field magnitudes in the adjacent media for the waves excited. The first term in Eq. (9) corresponds to Π_1 while the second term corresponds to Π_3. The partial energy flux within the vacuum gap is weak when the gap width is small,

$$\Pi_2 \sim O(hQ_2''). \tag{10}$$

Then the total energy flux $\Pi\equiv\Sigma_\ell\Pi_\ell\simeq\Pi_1+\Pi_3$. In Eqs. (9) and (10) the dispersion relation of the coupled waves, Eq. (5), is taken into account. Note that Eq. (9) is valid not only in the $-\varepsilon_3<\varepsilon_1(\omega)<0$ region, but this region is of specific interest.

For infinitely weak electron-beam density, $\eta_0\to 0$, $\omega''\to 0$, and Eq. (9) becomes

$$\Pi_1+\Pi_3 = \frac{c^2}{16\pi}|H_1^{(0)}(\omega,q)|^2\mathrm{Re}\left\{\frac{q}{\omega\varepsilon_3(\omega)Q_3''}\left[1-\frac{\varepsilon_3(\omega)^2}{\varepsilon_1(\omega)^2}\right]\right\} + O(hQ_2''). \tag{11}$$

Therefore, for the high-frequency-modified SPP mode the total energy flux is negative, $\Pi\simeq\Pi_1+\Pi_3<0$, for $-\varepsilon_3<\varepsilon_1(\omega)<0$. Note that in the vicinity of point B, $\omega\simeq\omega_{sp}$ and $\varepsilon_1(\omega)\simeq -1$ for $\varepsilon_3=1$ (cf. Fig. 2). From Eq. (11) it follows that the modules of the partial energy fluxes Π_1 and Π_3 are very close to each other, i.e., $|\Pi_1+\Pi_3|/\Pi_3\ll 1$. However, $\Pi_1+\Pi_3<0$. For a finite electron-beam density the total EMW energy flux is negative for the unstable mode ($\omega'' > 0$) in the vicinity of the intersection point B because the factor $|1+\eta_0|$ is less than unity.

Now we make a series of numerical calculations for the high-frequency region. For the widely used parameters, $\omega_0 = 5\times10^9$ s^{-1}, $\bar{\omega}_0 = 0.66$,[11] $N_0 = 10^{15}$ m^{-3}($\omega_B \approx 1.8\times10^9$ s^{-1}), $\beta = v_0/c = 0.1$, $\varepsilon_3 = \mu_1 = 1$, and $d \le h$, we find that for the unstable mode $\Pi \le 0$ at $h \le h_0$, where $h_0 \simeq 2.7\times10^{-4}$ m at $\lambda_B = 2\pi/q_B \simeq 3.5\times10^{-2}$ m. Specifically, for $h = 10^{-4}$ m$|1+\eta_0|^2 \simeq 0.96$ and $\Pi \simeq -0.22\Pi_3 < 0$. The increment for slow waves ($v_{\mathrm{ph}} = \omega/q \ll c$) is $\omega''_B \simeq 5\times10^{-2}\omega_P$ at $d = h = 10^{-4}$ m.

Note that the *quasisurface* waves with the negative total energy flux can exist in gaps between media other than those considered above. Specifically, there may be two metals, metal/dielectric, and two artificial media structures. The above results could be easily reformulated for some of these cases. For instance, for two neighboring metals the total energy flux is negative when the vacuum-gap width is smaller or equal to the skin depth, i.e., $h \le c/\omega_P \approx 5\times10^{-8}$ m for $\omega_P \simeq 5\times10^{15}$ s^{-1}. For other cases, the additional analysis is needed. A number of properties of these structures are discussed in Ref. 6.

Thus, in the present work the SPP waves excitation by the infinitely thin electron beam moving in the vacuum gap between the plasmalike medium and the artificial dielectric has been theoretically investigated. The dispersion relation for the coupled modes of an arbitrary vacuum-gap width has been found. The total energy flux is shown to be a negative one in the system with a finite vacuum-gap width in the presence of an electron beam.

The work was partially supported by STCU under Grant No. 3979.

*yuaver@online.kharkiv.com

[1] A. Schuster, *An Introduction to the Theory of Optics* (Arnold, London, 1904).
[2] L. I. Mandelstam, Zh. Eksp. Teor. Fiz. **15**, 475 (1945).
[3] The first name is due to the fact that the energy flux of these waves is codirected with the group velocity and thus antiparallel to the propagation direction given by $\mathbf{v}_{\mathrm{ph}}$. The second name reflects that the triplet given by the directions of the electric and magnetic fields along with the propagation direction $\mathbf{v}_{\mathrm{ph}}$ is left handed contrary to the right-handed triplet in vacuum or ordinary dielectric.
[4] H. Raether, *Surface Plasmons* (Springer, New York, 1988).
[5] R. Ruppin, Phys. Lett. A **277**, 61 (2000).
[6] N. Engheta, A. Alu, R. W. Ziolkowski, and A. Erentok, in *Metamaterials: Physics and Engineering Explorations*, edited by N. Engheta and R. Zioklowski (Wiley, New York, 2006), pp. 43–86.
[7] A. V. Kats, S. Savel'ev, V. A. Yampol'skii, and F. Nori, Phys. Rev. Lett. **98**, 073901 (2007).
[8] *Surface Polaritons*, edited by V. M. Agranovich and D. L. Mills (North-Holland, Amsterdam, 1982).
[9] Yu. O. Averkov and V. M. Yakovenko, Phys. Rev. B **72**, 205110 (2005).
[10] J. B. Pendry, A. J. Holden, W. J. Stewart, and I. Youngs, Phys. Rev. Lett. **76**, 4773 (1996).
[11] J. B. Pendry, A. J. Holden, D. J. Robbins, and W. J. Stewart, IEEE Trans. Microwave Theory Tech. **47**, 2075 (1999).
[12] D. R. Smith, W. J. Padilla, D. C. Vier, S. C. Nemat-Nasser, and S. Schultz, Phys. Rev. Lett. **84**, 4184 (2000).
[13] P. A. Sturrock, Phys. Rev. **112**, 1488 (1958).